The Search for Russian Alphamagic Squares

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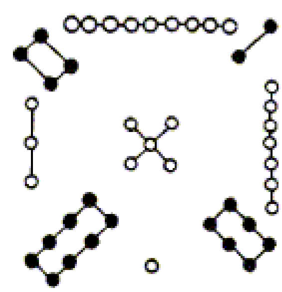
A **magic square** is a square array of numbers that add to a constant sum in any row, column, or diagonal. The oldest magic square known is called the ***Lo Shu*** (from the Chinese *shu*, meaning "writing," or "document"). It was reputed to have been revealed on the back of a *Lo* River turtle's shell to the mythical Emperor Yü in the 23rd Century B.C. The symbols on the back of the turtle's shell were represented as in figure 1.

Figure 1.



Notice that the dark symbols represent even values and the white symbols represent odd values, and that these values can be represented in our modern number convention as in figure 2.

Figure 2.

4	9	2
3	5	7
8	1	6

Here we see that any column, row, or diagonal of this array of numbers add to a constant sum of 15, thus making it a "magic" square. This "Lo Shu" was thought to have magical properties by the ancient Chinese and several other subsequent cultures as well. Symbols reflecting this array of numerical values were inscribed on amulets and charms in order to protect their owners from misfortune.

In the late seventeenth century the French mathematician Simon de la Loubère returned to France from his post as Ambassador to Siam with a method for the construction of any odd-order (i.e. 3 X 3, 5 X 5, 7 X 7…etc.) magic square with consecutive numbers. This "Siamese method" is difficult to describe, but involves beginning by placing the first number of a consecutive series in the center top cell of the array's grid. Then the process involves filling in the cell "up one and over one to the right" with the subsequent consecutive numbers. Let us begin an example by placing the number one (1) into the center top cell of a 3 X 3 grid. When the next number to be placed would fall off the grid, as indeed the first placement of the number two (2) would demand, then the number is instead placed at the extreme opposite end of the intended row or column. IF the number to be placed encounters either another number in its path OR would fall off a corner of the grid itself so that there is no opposite end of a row or column to place it in, THEN the number is placed into the cell immediately below the previous number and the process continues. An example of a third-order magic square created by this process is given in figure 3.

Figure 3.

8	1	6
3	5	7
4	9	2

This is a permutation, of course, of the *Lo Shu* and has a constant sum of 15. An example of a 7 X 7 magic square constructed by Loubére's "Siamese Method" is here in figure 4.

Figure 4.

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

The magic sum here is 175. The general formula for determining the magic sum of any order's magic square of consecutive numbers beginning with one (1) is $Ms=n^3 +n/2$ where n = the number of the square's order. So in the above 7 X 7 (order 7) square, $7^3 + 7 = 343 + 7 = 350/2 = 175$.

The first even-ordered magic square of prominence is that of the German Artist Albrecht Dürer (1471-1528) who included the following array of numbers on a grid in his engraving titled *Melencolia I.* This fourth order square is shown in figure 5.

Figure 5.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

The numbers of this amazing square add to 34 in any row, column, or full diagonal. The four corner values add to 34. The center four values add to 34. The four numbers in each of the four quadrants add to 34, and there are yet further symmetries. Moreover, the two center cells in the bottom row form the number 1514, signifying the year that Dürer engraved his *Melencolia I.*

Generations of recreational mathematicians have elaborated and refined the study of magic squares. The American renaissance man Benjamin Franklin was an avid creator of magic squares and wrote in his diaries that he had conceived a method that enabled him to create magic squares almost as fast as he could write. This included a well-published 8^{th} order magic square with numerous additional symmetries and an amazing 16 X 16 square with similar astounding symmetries of sum, but without equal sums on the main diagonals. Franklin reflected in his notes that the creation of magic squares was a pursuit "incapable of any useful application" except that it might train a mind to exactitude and diligence in other spheres. Modern experts on the creation of magic squares—scholars like Martin Gardner, Lee C. F. Sallows, Ivars Peterson, Harvey Heinz, Ian Stewart, John Robert Hendricks, Eric Weisstein, Clifford Pickover and others (see sources below), have had to contend with the accusation that their mathematical pursuits are not only recondite, but useless and without practicality. But this does not stop them and they remind us that much of current practicality was once considered recondite and useless. That is the nature of science and its progress.

In the early 1980's the Dutch mathematician, Lee C. F. Sallows, examining a mysterious book, published in 1887 and entitled *The Origin of Tree Worship*, encountered a relation about how a legendary Anglo-Saxon King of Northern England (Sallows records this King's name as "King Mi," but, not being able to find any other record of this king, we conjecture that it may have been King Ida (ruled ca. 550-571)) came to rule in the sixth century A.D. Taking the throne, this king instructed his young wizard, whose name is unfortunately lost to us, to provide him with a magic charm that would assure him of a long reign and a long life. He gave the wizard three days to come up with the charm. The wizard went into a nearby copse of yew trees for three days and, when he came out, he told the King that the desired charm was to be found on the bole of a yew tree in the center of the copse. Figure 6 represents what the King found inscribed in runes of the Anglo-Saxon Futhorc on the yew tree in the center of the copse, as well as Lee Sallows' first decipherment of these upside-down aettir (three rows) of runes into Old English words.

Figure 6.

Sallows immediately discerned that these Old English words could be written into modern English as figure 7. And that these words could be transcribed into numbers as in figure 8.

Figure 7.

Five	**Twenty-two**	**Eighteen**
Twenty-Eight	**Fifteen**	**Two**
Twelve	**Eight**	**Twenty-five**

Figure 8.

5	**22**	**18**
28	**15**	**2**
12	**8**	**25**

Sallows quickly realized that this is a magic square with a sum of 45 in any row, column, or diagonal. Surely this meant that the ancient runic inscription was intended by its mysterious creator to give his king advantages of its magic power. *But then Sallows recalled from his decipherment the number of the Old English letters (represented by the runes) needed to spell each number of the magic square.* Since the number of these letters (e.g. there are FOUR letters needed to spell the number 5, and EIGHT letters needed to spell the number 18,…and so on) do not differ from the Modern English (that is, *the* *number of runes or, subsequently, letters needed to spell the names of the numbers in English has not changed, despite the obvious historical changes in the number names themselves, for more than 1500 years*), they are written here as in figure 9.

Figure 9.

4	9	8
11	7	3
6	5	10

Notice that the hyphens in "twenty-two," "twenty-five," and "twenty-eight" are not counted (since they weren't there in Old English as reflected in the runes). *But this too is a magic square*, having all rows, columns, and diagonals sum to 21. **What we have here is an extraordinary confluence of "magic" in the world of numbers with "magic" in the world of words**. This first runic square is doubly magical. It is, as Sallows termed it when he published his analysis of it in 1986, an **ALPHAMAGIC SQUARE**, the first one known. Our conjecture is that the young wizard, age 21, conceived it as a magic charm for his king (King Ida?), who was 45 years of age, and that the charm worked such magic for the king that his rule and his life extended to the very ripe old age (for those times) of 66 years, the sum of his age and the age of his wonderful wizard (note that the years of King Ida's rule are given as 550-571 A.D.).

In two now classic articles published in the journal Abacus (Vol. 4, No. 1 (Fall 1986), 28-45, and Vol. 4, No. 2 (Winter 1987), 20-29), Sallows investigates the "Alphamagic Squares." He writes: "*Alphamagic* is the word I use to describe any magic array…that remains magic when all of its entries are replaced by numbers representing the word length, in letters, of their conventional written names (thus, one is replaced by 3, since it takes three English alphabet letters to spell the name of the number 1 (one))." Sallows then describes a process by which alphamagic squares may be discovered. He defines a "Logorithm" (from Greek root "Logos" for the "word," and "Arithmos" for "number" and as distinct from the more conventional mathematical "Logarithm") as "the letter count of the number word." This means that the logorithm of twenty-eight is 11 because there are eleven letters in the spelling of the number 28's name. Trying to find a third-order alphamagic square in English, he then places the consecutive numbers in a row above each number's respective logorithm. In this array he seeks constant-difference triples of both the numbers and the logorithms that center on a single axis. This is because there is always a constant difference between the three numbers of any line of numbers passing through the center of a magic square. As Ian Stewart describes it: "So a reasonable strategy for finding alphamagic squares is to look for triples of numbers in arithmetic series, such that the corresponding sequence of logorithms also forms an arithmetic series." A partial example is given here in figure 10.

Figure 10.

Numbers:..	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22…
Logarithms:.	5	4	3	6	6	8	8	7	7	9	8	8	6	9	9…

A study of these correspondences reveals some likely triples in arithmetic series: 8 plus 7 to 15 plus 7 to 22 (8, 15, 22 with constant difference of 7) in the numbers, and the corresponding 5 plus 2 to 7 plus 2 to 9 (5, 7, 9 with constant difference of 2) in the logorithms. Also one can find the number triple of 12, 15, 18 (constant difference of 3) corresponding to the logorithm triple of 6, 7, 8 (constant difference of 1). These two corresponding sets of constant-difference triples center on the number 15, so an attempt is made with 15 in the center in figure 11.

Then we add the other triple on the other diagonal, centered on the center value of 15, as in figure 12. The other numbers fall into place by mathematical necessity since the magic sum clearly has to be 45. The resultant magic square of numbers is then given in figure 13.

Figure 11.

8		
	15	
		22

Figure 12.

8		18
	15	
12		22

Figure 13.

8	19	18
25	15	5
12	11	22

And to test to see if this is not only a magic square, but an alphamagic square, all we have to do is examine the array of the corresponding logorithms, given here in figure 14.

Figure 14.

Eight (5 letters)	Nineteen (8 letters)	Eighteen (8 letters)
Twenty-five (10 letters)	Fifteen (7 letters)	Five (4 letters)
Twelve (6 letters)	Eleven (6 letters)	Twenty-two (9 letters)

And yes, this array of the English logorithms is itself a magic square, with magic sum of 21 in all ways. SO, the above third-order magic square of numbers 8, 19, 18 / 25, 15, 5 / 12, 11, 22 IS an alphamagic square. Hoorah, we've found one by a systematic procedure!

Making use of the general formula for a third-order magic square put forth by French number theorist Edouard Lucas (1842-1891) in figure 15, and consulting with his colleague Victor Eijkhout, Sallows was able to come up with a computer program in ALPHA.BAS, "a Pascal form," to generate values of "A" that allow for coincidences of constant-difference triples among the numbers and logorithms of English. Using this program he has found numerous other alphamagic squares in English. Confining himself just to magic squares of all-different numerical values from 1-100, he has published "the first ten English alphamagic squares of order 3." The English alphamagic square we constructed above as a demonstation is number two, after number

Figure 15.

A + B	A – B - C	A + C
A – B + C	A	A + B – C
A - C	A + B + C	A - B

one, that first one found on the yew tree written in runes, the one Sallows has named ***Li Shu*** (n.b.—not “Lo Shu” like the one on the turtle in China).

Sallows extended his computerized procedure to the finding of alphamagic squares in other languages, and he found and published many. He even defined there to be “translation” relationships of alphamagicality between his discovered squares. Either two different primary magic squares could share the same secondary logorithmic magic square. Or one primary magic square could be alphamagical in more than one language, the two secondary logorithmic squares, though different, both being magic. He pointed out that the concept of alphamagic squares applies to all the alphabetic languages, writing, “Clearly there are many. Besides those like our own employing Roman letters, *there remain others using the Greek, Hebrew, and Cyrillic alphabets* (italics here ours). The work of collecting and collating alphamagic squares in the various tongues and dialects opens a wide (if decidedly recondite) area of research.”

Sallows’ inventory of published alphamagic squares includes nineteen languages including Swahili, Samoan, Gaelic, Welsh, Esperanto, Turkish, Finnish, Maltese, and others more widely spoken. **Inspired by Sallows’ work, and not being able to find any published alphamagic squares in the Russian Cyrillic after more than twenty years of possibility, Arizona State University Slavic linguist, Professor Lee B. Croft, set about to see if he could discover the first Russian Alphamagic Square**.

Croft’s first attempts were to check all the alphamagic squares in the diverse languages using the “Roman letters” that were published by Lee Sallows to see if any were also alphamagical in the Russian Cyrillic. None of them were.

Croft then listed the Russian numbers from 1-150 and counted the numbers of letters needed to spell them in the Cyrillic alphabet. He then placed all the numbers in the constituency of each “logorithm” into tables and checked to see if any third or fourth-order magic squares could be made from the numbers belonging to any one logorithm. This investigation resulted in two encouraging “trivial” cases of Russian Cyrillic alphamagic squares. The first is the result of the fact that the Russian numbers 8, 9, and 10, are the only consecutive numbers (thus having a constant difference of one (1)) all of which are in the constituency of the logorithm 6 (i.e. the names of these numbers, “**восемь, девять, десять**” all have six (6) letters in their spelling). So, then, figure 16 shows this “trivial” (having repeated numbers in its cells) magic square, having a magic sum of 27. And this 3 X 3 array has the corresponding array of logorithms shown in figure 17.

Figure 16.

9	8	10
10	9	8
8	10	8

Figure 17.

6	6	6
6	6	6
6	6	6

This is an array with a magic sum of 18, thus proving the third-order magic square of 9, 8, 10 / 10, 9, 8 / 8, 10, 9 to be a Russian Cyrillic alphamagic square, albeit a “trivial one.”

Similarly, there are only four Russian Cyrillic numbers in the constituency of logorithm 4. These are “**ноль** (0), **один** (1), **пять** (5), and **семь** (7).” Unlike the third-order square, which needs a constant difference between each of the three numbers in its lines through the center, a fourth-order “trivial” magic square may be constructed of *any* four numbers. So, the following square in figure 18, having the magic sum 13, has the

Figure 18.

0	1	5	7
7	5	1	0
1	0	7	5
5	7	0	1

Figure 19.

4	4	4	4
4	4	4	4
4	4	4	4
4	4	4	4

underlying array of logorithms shown in figure 19, and this array has a magic sum of 16, thus proving the fourth-order magic square of 0, 1, 5, 7 / 7, 5, 1, 0 / 1, 0, 7, 5 / 5, 7, 0, 1 also to be a Russian Cyrillic alphamagic square, albeit a "trivial one."

A similar "trivial" fourth-order alphamagic square in Russian Cyrillic can be constructed from a set of four numbers all sharing the logorithm 14, as in figure 20.

Figure 20.

24 (14) Двадцать четыре	61 (14) Шестьдесят один	76 (14) Семьдесят шесть	111 (14) Сто одиннадцать
111 (14) Сто одиннадцать	76 (14) Семьдесят шесть	61 (14) Шестьдесят один	24 (14) Двадцать четыре
61 (14) Шестьдесят один	24 (14) Двадцать черыре	111 (14) Сто одиннадцать	76 (14) Семьдесят шесть
76 (14) Семьдесят шесть	111 (14) Сто одиннадцать	24 (14) Двадцать четыре	61 (14) Шестьдесят один

Here the magic sum of the numbers is 272 and of the associated logorithms 56.

After much trial and error, Croft was able to find what recreational mathematicians call a third-order "*semimagic square*," that is, a square of magic sum of all columns and rows, but with a variance of one (and by other definitions two, which may or may not be the same) diagonal. This is shown in figure 21.

Figure 21.

74	30	62
60	72	34
32	64	70

Figure 22.

Семьдесят четыре (logo 15)	Тридцать (logo 8)	Шестьдесят два (logo 13)
Шестьдесят (logo 10)	Семьдесят два (logo 12)	Тридцать четыре (logo 14)
Тридцать два (logo 11)	Шестьдесят четыре (logo 16)	Семьдесят (logo 9)

This array sums to 166 in any column, row, and the right-top-to-left-bottom diagonal, but is only "semimagic" because its numbers in the left-top-to-bottom-right diagonal add not to 166, but to 216 (this should have been a hint…see below). BUT, if we write these numbers out in the Russian Cyrillic we get the array shown in figure 22.

The logorithmic values of this array are pleasingly different in each cell (consecutive 8-16) and have a magic sum of 36 in each column, row, and both diagonals. SO, Croft discovered by trial-and-error a third-order semimagic square that is alphamagic in Russian Cyrillic.

Similarly by trial and error, Croft came up with a "close miss" on a fourth-order square in which every cell has a different number. This is in figure 23.

In this array, all the numerical columns, rows, and one diagonal sum to 255. But the right-top-to-bottom-left diagonal sums to 251…requiring the bottom left cell to be 75. This would result in that cell having the same logorithm of 13. The sums of all the logorithms show that cells nine (left on third row) and fourteen (bottom of second column) are defective of allowing a magic sum of 54, and need to be, respectively 15 and 12 as noted above. So this then is a semimagic square that is almost alphamagical in Russian Cyrillic. Close, but no cigar, as they say.

Figure 23.

19 (12) Девятнадцать	**62 (13) Шестьдесят два**	**76 (14) Семьдесят шесть**	**98 (15) Девяносто восемь**
83 (14) Восемьдесят три	**94 (15) Девяносто четыре**	**21 (12) Двадцать один**	**57 (13) Пятьдесят семь**
82 (14...15) Восемьдесят два	**61 (14) Шестьдесят один**	**77 (13) Семьдесят семь**	**35 (12) Тридцать пять**
71 (13)/75 Семьдесят один	**38 (14...12) Тридцать восемь**	**81 (15) Восемьдесят один**	**65 (14) Шестьдесят пять**

As Croft was unable to make the ALPHA.BAS "Pascal" program published by Lee Sallows work (likely Lee Croft's fault), he approached Samuel Comi, an Arizona State University Mathematics major, for advice. Comi readily understood the problem and gave his opinion that he could write a program in Java that would use the Lucas formula to generate third-order alphamagic squares if given the Sallows' logorithms to correlate with the Russian Cyrillic numbers. In a single weekend, Comi came up with the Java program (see appendix) and, shown here in figure 24, the first non-trivial Russian alphamagic square.

Figure 24.

74 (15) Семьдесят Четыре	**50 (9) Пятьдесят**	**92 (12) Девяносто Два**
90 (9) Девяносто	**72 (12) Семьдесят Два**	**54 (15) Пятьдесят Четыре**
52 (12) Пятьдесят Два	**94 (15) Девяносто Четыре**	**70 (9) Семьдесят**

The numbers in this third-order square add to 216 in any row, column, or diagonal, while the associated logorithms (the number of Cyrillic-alphabet letters required to spell the name of each Russian number) add to 36 in any row, column, or diagonal. **Therefore, the third-order square of 74, 50, 92 / 90, 72, 54 / 52, 94, 70 is a Russian Cyrillic alphamagic square…<u>the first non-trivial one discovered</u>**. Moreover, the computer program shows this to be the *only one* with numbers from one (1=**Один**) to a hundred (100 = **Сто**). Authors Lee Croft and Samuel Comi call it, in accordance with the tradition established by the earlier "Lo Shu" and "Li Shu," the "**Lee Sam**," which is also a pun on Lee Croft's first-name Russian signature, "**Ли сам**," pronounced "Lee Sam" and meaning "Lee himself" in Russian. In addition to its alphamagic property, the "Lee Sam" has some other interesting properties. First is the property that the separated digits of its constituent numbers are themselves magical. That is, the first digits of the numbers 74 (i.e. 7), 50 (i.e. 5), 92 (i.e. 9) / 90 (i.e. 9), 72 (i.e. 7), 54 (i.e. 5)/ 52 (i.e. 5), 94 (i.e. 9), and 70 (i.e. 7), thus forming an array of 7, 5, 9/9, 7, 5 / 5, 9, 7 form a magic square where the rows, columns and diagonals add to a constant sum of 21. And, the second digits of the constituent numbers 74 (i.e. 4), 50 (i.e. 0), 92 (i.e.2) / 90 (i.e. 0), 72 (i.e. 2), 54 (i.e. 4) / 52 (i.e. 2), 94 (i.e. 4), 70 (i.e. 0), thus forming an array of 4, 0, 2/ 0, 2, 4 / 2, 4, 0, form a magic square where the rows, columns and diagonals add to a constant sum of 6. This "divisional digit" property means also that the "digital reversal" array of 47 (from 74), 05 or 5 (from 50), 29 (from 92) / 09 or 9 (from 90), 27 (from 72), 45 (from 54) / 25 (from 52), 49 (from 94), 07 or 7 (from 70) forms a magic square whose constant sum on any row, column or diagonal is 81, three times the central cell's digitally reversed value of 27. Neither of these divisional digit squares nor the digital reversal square is alphamagical, however, in either English or Russian.

It turns out that the only other alphamagic square in Russian Cyrillic with non-repeating numbers less than 200 can be constructed merely by adding the digit 1-, signifying 174, 150, 192 / 190, 172, 154 / 152, 194, 170, the numbers of the above square each increased by a hundred (i.e. adding the Russian word for 100, or "**Сто**," with logorithm of 3). Lee Sallows has termed such an alphamagic square a "second harmonic" of the first. But, if we expand the search into higher numbers there are hundreds of others. The fact that the Russian word for 200 ("**Двести**") has a logorithm of 6—double the logorithm of the word for 100, means that numerous triples of constant difference can be found wherein a sub-100 number is combined with a 100+that number and a 200+that number to form the lines through the center of a third-order square. An example of one of these is given in figure 25.

Figure 25.

154 (18) **Сто пятьдесят четыре**	**48 (11)** **Сорок Восемь**	**251 (19)** **Двести Пятьдесят Один**
248 (17) **Двести Сорок Восемь**	**151 (16)** **Сто пятьдесят Один**	**54 (15)** **Пятьдесят Четыре**
51 (13) **Пятьдесят Один**	**254 (21)** **Двести Пятьдесят Четыр**	**148 (14)** **Сто Сорок Восемь**

The magic sum of the numbers of this third-order Russian Cyrillic alphamagic square is 453 in any row, column, or diagonal, and the magic sum of the related logorithms (all pleasingly different) is 48. Other even higher numbered squares are likely to result in similar fashion from the fact that the Russian number for 400 ("**четыреста**") has a logorithm of 9, thus making possible numerous triples of concentric constant difference numbers less than a hundred, with 200+numbers (adding 6 to the logorithm) and 400+numbers (adding 9 to the logorithm). As the number triples increase by 200, the concentric logorithms increase by 3—fertile ground to find alphamagic squares.

Following Sallows' lead in the matter of alphamagic square "translations," Croft began an investigation to see if he could find other languages in which the magic square of 74, 50, 92 / 90, 72, 54 / 52, 94, 70 is alphamagical—thus finding a "translation" of the "Lee Sam" Russian Cyrillic alphamagic square into another language. He thought initially to find a translation between a language using the Roman-based alphabet (like English and all the others found by Sallows in his initial articles) and the Russian Cyrillic. So, he looked up the sets of numbers in diverse Roman-alphabet languages in online compendia and counted the numbers of letters to spell these other languages' words for these numbers, checking to see if the resultant logorithmic squares were magical. It was surprising how many of the logorithmic squares had constant-sum rows and columns. The great majority are, in fact, semimagic, but miss having either one diagonal or both diagonals in accord with the prevailing constant sum.

These *three* Roman-alphabet languages have their logorithmic squares on the numbers of the "Lee Sam" Russian Cyrillic alphamagic square with only one diagonal at variance from magical (i.e. the rows, columns, and one diagonal add successfully to the same constant sum): *German, Italian and Polish*. So, there is no true translation here.

These *twenty* Roman-alphabet languages have their logorithmic squares on the numbers of the "Lee Sam" Russian Cyrillic alphamagic square with both diagonals at variance from magic (i.e. the rows and columns add successfully to the same constant sum, but both diagonals are different): *Basque, Czech, English, Estonian, Finnish, French, Hawaiian, Latvian, Lithuanian, Nahuatl, Portuguese, Romanian, Seneca, Spanish, Swahili, Swedish, Turkish, Tzotzil, Welsh, and Zulu*. There is also no true translation here.

An attempt was made to check the Roman-alphabetic rendering of the "Lee Sam's" numbers of *Navajo* to see if the secondary logorithms are magic. Two columns and two rows add to 58, one column and one row add to 60, one diagonal adds to 60 and the other to 73. *Navajo* clearly offers no translation of the "Lee Sam" here.

Trying the other-alphabet languages of *Armenian* (rows and columns add to the same constant sum, but both diagonals are different) and *Georgian* (not even the rows and columns add to a constant sum) does not find a true translation of the "Lee Sam" Russian Cyrillic alphamagic square.

In ancient *Greek* and *Hebrew* the situation is the same. The numbers 50, 70, and 90 each take only one letter to represent. The numbers 52, 54, 72, 74, 92, and 94 each take two. So the resultant logorithmic square is 2, 1, 2 / 1, 2, 2 / 2, 2, 1 for both ancient *Greek* and *Hebrew*—both just the right-top-to-left-bottom diagonal different. There is no true translation here either.

Giving up on the other-alphabet languages and trying the related Cyrillic-alphabet Slavic Language of *Bulgarian* (a South Slavic congener) finds both diagonals of the logorithmic square different. And even the closely related East Slavic language of *Ukrainian* finds the secondary logorithmic square missing a constant sum on one diagonal. SO, this is 30 languages with diverse alphabets checked without finding a proper translation. **So far, therefore, a true translation of this "Lee Sam," the first third-order Russian Cyrillic Alphamagic Square has yet to be found.** It's interesting to speculate that it may be untranslatable among the world's alphabetic languages, but the search is not over.

In his original work, Lee Sallows discussed the possibility of finding higher-order alphamagic squares, that is, 4 X 4, or 5 X 5, or higher. He wrote: "With the transition from order 3 to order 4, and higher, comes a concomitant jump in the perplexities confronting our advance, since hindsight reveals 3 as a special, unusually tractable case." Even-order magic squares are more complex to derive than odd-order ones, and programming a computer to help in the task is significantly more difficult. Yet as he discusses these difficulties in his "Alphamagic Squares" article, Ivars Peterson asks: "…are there any instances of four-by-four and five-by-five language-dependent alphamagic squares?" And, surprisingly and without explanation of process, he answers: "A quick search turns up several examples. The following table of numerical values is an example of a four-by-four alphamagic square in English." Peterson's 4 X 4 English alphamagic square, published for the first time, apparently, in 2003, is shown in figure 26.

Figure 26.

26	37	48	59
49	58	27	36
57	46	39	28
38	29	56	47

Now this is a really astonishing array of numbers. It is not a perfect pan-diagonal magic square, but it has a number of amazing symmetries. It is a conventional magic square because all its rows, columns and main diagonals add to a constant sum of 170. But notice that its four corner values add to 170. Its four central values add to 170. If we divide the whole square into its four constituent quadrants, then the numbers of each quadrant add to the constant sum of 170. Both sets of 2-2 broken diagonals (i.e. 49, 37, 56, and 28, and also 57, 29, 48, and 46) add to the constant sum of 170. Only the 3-1 diagonals and the central horizontal and central vertical quadrants are divergent to sums of 210 or 130 complementarily. It is only that shy of perfection.

To see that this magic square is alphamagical in English, we need to spell out the English numbers in the Roman alphabet and record the count of letters as in figure 27.

Figure 27.

Twenty-six (9)	**Thirty-seven (11)**	**Forty-eight (10)**	**Fifty-nine (9)**
Forty-nine (9)	**Fifty-eight (10)**	**Twenty-seven (11)**	**Thirty-six (9)**
Fifty-seven (10)	**Forty-six (8)**	**Thirty-nine (10)**	**Twenty-eight (11)**
Thirty-eight (11)	**Twenty-nine (10)**	**Fifty-six (8)**	**Forty-seven (10)**

The secondary array here of the numbers of Roman-alphabet letters to spell the English names of the primary numbers does add to a constant sum of 39 in any row, column, or main diagonal. So Peterson's 4 X 4 array does check out to be alphamagical in English.

Another interesting property of Peterson's 4 X 4 array is that it, like the "Lee Sam" 3 X 3, remains a magic square under the "divisional digits" and the "reversed digits" permutations. That is, the digitally reversed 4 X 4 of 62 (from 26), 73 (from 37), 84 (and so on…), 95 / 94, 85, 72, 63 / 75, 64, 93, 82 / 83, 92, 65, 74 adds to a constant sum of 314 on any row, column, or main diagonal. AND, unlike the "Lee Sam," this digitally reversed magic square is proven to be alphamagical in English as well, since its secondary array, formed from the number of Roman-alphabet letters to spell the English numbers in it, (i.e. sixty-two=8, seventy-three=12, …10, 10 / 10, 10, 10, 10 / 11, 9, 11, 9 /

11, 9, 9, 11) adds to 40 on any row, column, or main diagonal (note that the constant sum (40) of the digitally reversed square's secondary square is different than the constant sum (39) of the primary square's secondary square). This would then be the first demonstration of an alphamagic square remaining alphamagical with its numbers digitally reversed.

We now need to ask whether this impressive 4 X 4 array, alphamagical in *English*, is alphamagical as well in other *Roman*-alphabet languages. Can we find a translation of it? Let's begin by trying *German*. In *German* we can spell the numbers of Peterson's array as shown in Figure 28.

Figure 28.

26 (15) **Sechsundzwanzig**	**37 (17)** **Siebenunddreissig**	**48 (14)** **Achtundvierzig**	**59 (14)** **Neunundfünfzig**
49 (14) **Neunundvierzig**	**58 (14)** **Achtundfünfzig**	**27 (16)** **Siebenundzwanzig**	**36 (16)** **Sechsunddreissig**
57 (16) **Siebenundfünfzig**	**46 (15)** **Sechsundvierzig**	**39 (15)** **Neununddreissig**	**28 (14)** **Achtundzwanzig**
38 (15) **Achtunddreissig**	**29 (14)** **Neunundzwanzig**	**56 (15)** **Sechsundfünfzig**	**47 (16)** **Siebenundvierzig**

And here we can add the rows, columns, and both main diagonals of the logorithms to 60, a constant sum. We have therefore found a *German* alphamagic square, the translation of Ivars Peterson's 4 X 4 English alphamagic square. In French, this array becomes figure 29.

Figure 29.

26 (8) **Vingt-six**	**37 (10)** **Trente-sept**	**48 (12)** **Quarante-huit**	**59 (13)** **Cinquante-neuf**
49 (12) **Quarante-neuf**	**58 (13)** **Cinquante-huit**	**27 (9)** **Vingt-sept**	**36 (11)** **Trente-six**
57 (13) **Cinquante-sept**	**46 (11)** **Quarante-six**	**39 (10)** **Trente-neuf**	**28 (9)** **Vingt-huit**
38 (10) **Trente-huit**	**29 (9)** **Vingt-neuf**	**56 (12)** **Cinquante-six**	**47 (12)** **Quarante-sept**

The French logorithms add to 43 in any row, column, or main diagonal, making this a French 4 X 4 alphamagic square, a translation of Ivars Peterson's original 4 X 4 *English* alphamagic square and/or of the *German* example above. Spanish appears in Figure 30.

In the *Spanish* example of Figure 11 the logorithms add to 51 in any row, column, or main diagonal. So here we also have a Spanish 4 X 4 alphamagic square, and another translation. Do there exist any Roman-alphabet translations from languages less closely related to these?

Figure 30.

26 (10) **Veintiseis**	**37 (13)** **Treinte y siete**	**48 (13)** **Cuaranta y ocho**	**59 (15)** **Cinquanta y nueve**
49 (14) **Cuaranta y nueve**	**58 (14)** **Cincuenta y ocho**	**27 (11)** **Veintisiete**	**36 (12)** **Treinte y seis**
57 (15) **Cincuenta y siete**	**46 (13)** **Cuaranta y seis**	**39 (13)** **Treinta y nueve**	**28 (10)** **Veintiocho**
38 (12) **Treinta y ocho**	**29 (11)** **Veintinueve**	**56 (14)** **Cincuenta y seis**	**47 (14)** **Cuaranta y siete**

Here is the Hungarian array in Figure 31.

Figure 31.

26 (9) **Huszonhat**	**37 (10)** **Harminchet**	**48 (12)** **Negyvennyolc**	**59 (11)** **Otvenkilenc**
49 (13) **Negyvenkilenc**	**58 (10)** **Otvennyolc**	**27 (9)** **Huszonhet**	**36 (10)** **Harminchat**
57 (8) **Otvenhet**	**46 (10)** **Negyvenhat**	**39 (13)** **Harminckilenc**	**28 (11)** **Huszonnyolc**
38 (12) **Harmincnyolc**	**29 (12)** **Huszonkilenc**	**56 (8)** **Otvenhat**	**47 (10)** **Negyvenhet**

Since the logorithms here add to a constant sum of 42 on any row, column, or main diagonal, we also have here a *Hungarian* 4 X 4 alphamagic square, and yet another flawless translation. Notice that the constant sums of the logorithms are different for English, German, French, Spanish, and Hungarian. All are alphamagical in different ways.

We now have a 4 X 4 array of numbers with wondrous properties of symmetry that has proven to be magical and alphamagical in English. It has remained magical and alphamagical in English when its every number is digitally reversed. It is alphamagical not only in English, but also in the other Roman-alphabet languages of German, French, Spanish, and even less closely related Hungarian. By now one might wonder if there are any Roman-alphabet languages in which this array is NOT alphamagical. I assure you that there are many…Latin, for example, Hawaiian, and others.

But now we must try to see if there might be a translation of this polylingual alphamagic square into a non-Roman-alphabet language. Here is the Russian Cyrillic-alphabet version of this fourth-order array in figure 32.

Figure 32.

26 (13) Двадцать Шесть	37 (12) Тридцать Семь	48 (11) Сорок Восемь	59 (15) Пятьдесят Девять
49 (11) Сорок Девять	58 (15) Пятьдесят Восемь	27 (12) Двадцать Семь	36 (13) Тридцать Шесть
57 (13) Пятьдесят Семь	46 (10) Сорок Шесть	39 (14) Тридцать Девять	28 (14) Двадцать Восемь
38 (14) Тридцать Восемь	29 (14) Двадцать Девять	56 (14) Пятьдесят Шесть	47 (9) Сорок Семь

And now we can see the discovery of the first non-trivial (see Croft and Comi in "Sources" for a trivial one) fourth order (4 X 4) Russian alphamagic square and the first non-Roman-alphabet (i.e. Cyrillic alphabet) fourth-order alphamagic square. The constant sum of all the rows, columns, and main diagonals of the array of the number of Cyrillic letters in the Russian number names is 51 (identical in secondary sum to the Spanish example in figure 11, though the cells are not the same). And, as unbelievable as it may seem, the digitally reversed version of this array is also alphamagical in Russian Cyrillic, as seen in figure 33.

Figure 33.

62 (13) Шестьдесят Два	73 (12) Семьдесят Три	84 (17) Восемьдесят Четыре	95 (13) Девяносто Пять
94 (15) Девяносто Четыре	85 (15) Восемьдесят Пять	72 (12) Семьдесят Два	63 (13) Шестьдесят Три
75 (13) Семьдесят Пять	64 (16) Шестьдесят Четыре	93 (12) Девяносто Три	82 (14) Восемьдесят Два
83 (14) Восемьдесят Три	92 (12) Девяносто Два	65 (14) Шестьдесят Пять	74 (15) Семьдесят Четыре

Here we see that the constant sum of all rows, columns, and main diagonals is 55 (not the same as in the original's secondary of 51, but evidencing the digital-reverse array's Russian alphamagicality). And now, in addition to the English version investigated earlier, we have a fourth-order Russian alphamagic square that remains alphamagical in Russian when its numbers are digitally reversed. We are reminded of the poet William Blake's wonderment in apprehension of the "Tyger:" "What immortal hand or eye, could frame thy fearful symmetry?" **Indeed this array is a multiply magical wonder, *polylingually, bialphabetically, and digital-reversably alphamagical…***

In what other languages and other alphabets might this fourth-order magic square prove to be alphamagical? *What other worlds of words might these numbers bridge?*

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Appendix

Samuel Comi's Java Program for finding 3 X 3 Alphamagic Squares in Russian

```
//Samuel Comi
//14 Feb 2008
//This program will find 3x3 Alphamagic squares within
//the chosen limit, (up to 1000) for the Russian language and may be
//adapted to find other alphabetic languages' alphamagic squares by
//using different languages' Sallows' "logorithms" (see text).
import java.util.*;
public class Logorithm
{
   public static void main (String[] args)
   {
      //Russian
      int[] zero = new int[10];
      zero[0] = 0;
      zero[1] = 4;
      zero[2] = 3;  //These establish the values
      zero[3] = 3;  //for number length less than
      zero[4] = 6;  //10
      zero[5] = 4;
      zero[6] = 5;
      zero[7] = 4;
      zero[8] = 6;
      zero[9] = 6;
      int[] ten = new int[10];
      ten[0] = 6;
      ten[1] = 11;
      ten[2] = 10;
      ten[3] = 10;  //These establish the values
      ten[4] = 12;  //for numbers from 10 through 19
      ten[5] = 10;
      ten[6] = 11;
      ten[7] = 10;
      ten[8] = 12;
      ten[9] = 12;
```

```
      int[] sto = new int[100];
      for (int i = 0; i < 100; i ++)
      {       //This loop establishes the values
         if (i < 10)    //for numbers up to 100
         sto[i] = zero[i];
         else{ if ( i < 20)
            sto[i] = ten[i % 10];
            else{ if (i < 40)
               sto[i] = zero[i % 10] + 8;
               else{ if (i < 50)
                  sto[i] = zero[i % 10] + 5;
                  else{ if (i > 59 && i < 70)
                     sto[i] = zero[i % 10] + 10;
                     else{ if (i > 79 && i < 90)
                        sto[i] = zero[i % 10] + 11;
                        else sto[i] = zero[i % 10] + 9;
      }  } } } } }
      int[] thousand = new int[1000];
      for (int i = 0; i < 1000; i ++)
      {       //This loop establishes the values
         if (i < 100)   //for numbers up to 1000
            thousand[i] = sto[i];
            else{ if (i < 200)
               thousand[i] = sto[i % 100] + 3;
               else{ if (i < 400)
                  thousand[i] = sto[i % 100] + 6;
                  else{ if ((i > 499 && i < 600) || (i > 699 && i <
800))
                     thousand[i] = sto[i % 100] + 7;
                     else{ if (i > 599 && i < 700)
                        thousand[i] = sto[i % 100] + 8;
                        else thousand[i] = sto[i % 100] + 9;
      }   } } } }
            //Query user for limiting value
      System.out.print("What is the limiting value?");
            //Reads input value
      Scanner input = new Scanner (System.in);
      int max = input.nextInt();
            //Sets limit at 1000 if user put a larger value
      if (max > 1000)
      max = 1000;
            //creates array with all pertinent logorithm values
      int[] log = new int[max];
      for (int i = 0; i < max; i++)
         log[i] = thousand[i];
```

```
         //loop examines all possible triples within the
         //prescribed perameters capable of defining a
         //magic triangle once, then tests the chosen triangle
         //for alphamagicity.
   for (int i = 3; i < max / 2; i++)
   {
      for (int j = 1; j < i - 1; j++)
      {
         for (int k = j + 1; k < i; k++)
         {
            test(i, j, k, log, max);
            test(max - i, j, k, log, max);
         }
      }
   }
}
//Method tests the alphamagicity of the magic square indicated by
//the first three values, based on the array of logorithmic
//values and the limiting value passed to the method.
public static void test(int i, int j, int k, int[] log, int max)
{
   //instantiation of logorithmic values, and definition
   //of values in the magic square based on the defining
   //values passed to the method
   int sum, l, m, n, o, p, q, r, s, t;
   int aa = i + j;
   int ab = i - j - k;
   int ac = i + k;
   int ba = i - j + k;
   int bb = i;
   int bc = i + j - k;
   int ca = i - k;
   int cb = i + j + k;
   int cc = i - j;
      //checks to make sure the values of the magic square
      //are within prescribed parameters
   if (ab < max && ab > 0 &&
      ba < max && ba > 0 &&
      bc < max && bc > 0 &&
      ca < max && ca > 0 &&
      cb < max && cb > 0 &&
      cc < max && cc > 0 )
   {
      //definition of logorithmic squares
      l = log[aa];
```

```
         m = log[ab];
         n = log[ac];
         o = log[ba];
         p = log[bb];
         q = log[bc];
         r = log[ca];
         s = log[cb];
         t = log[cc];
         sum = l + m + n;
           //checks whether the square with logorithmic values is magic
         if (l + m + n == sum &&
            o + p + q == sum &&
            r + s + t == sum &&
            l + p + t == sum &&
            n + p + r == sum &&
            l + o + r == sum &&
            m + p + s == sum &&
            n + q + t == sum)
            //prints out the three defining values of the square that
            //has passed all of the tests
            System.out.println(i + ", " + j + ", " + k);
      }
   }
}
/*   //English
     //alternate creation of the array "thousand" used to establish
     //logorithmic values
      int[] zero = new int[10];
      zero[0] = 0;
      zero[1] = 3;
      zero[2] = 3;
      zero[3] = 5;
      zero[4] = 4;
      zero[5] = 4;
      zero[6] = 3;
      zero[7] = 5;
      zero[8] = 5;
      zero[9] = 4;
      int[] ten = new int[10];
      ten[0] = 3;
      ten[1] = 6;
      ten[2] = 6;
      ten[3] = 8;
      ten[4] = 8;
      ten[5] = 7;
      ten[6] = 7;
```

```
ten[7] = 9;
ten[8] = 8;
ten[9] = 8;
int[] hundred = new int[100];
for (int i=0; i < 100; i++)
{
   if (i < 10)
   hundred[i] = zero[i];
   else{if (i < 20)
      hundred[i] = ten[i % 10];
      else{if (i < 40 || i > 79)
         hundred[i] = 6 + zero[i % 10];
         else{if (i < 70)
            hundred[i] = 5 + zero[i % 10];
            else{if (i < 80)
               hundred[i] = 7 + zero[i % 10];
} } } } }
int[] thousand = new int[1000];
for (int i=0; i < 1000; i++)
{
   if (i < 100)
   thousand[i] = hundred[i % 100];
   else{if (i < 300 || (i > 599 && i < 700))
      thousand[i] = hundred[i % 100] + 10;
      else{if (i < 400 || (i > 699 && i < 900))
         thousand[i] = hundred[i % 100] + 12;
         else
         thousand[i] = hundred[i % 100] + 11;
      }
   }
}
*/
```

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